AD-A035 353

FLORIDA STATE UNIV TALLAHASSEE DEPT OF STATISTICS F/G 12/1
UNIFIED TREATMENT OF INEQUALITIES OF THE WEIERSTRASS PRODUCT TY--ETC(U)
DEC 76 E EL-NEWEIHI, F PROSCHAN AF-AFOSR-2581-74
FSU-STATISTIC-M396 AFOSR-TR-77-0034 NL

UNCLASSIFIED

OF | AD A035353











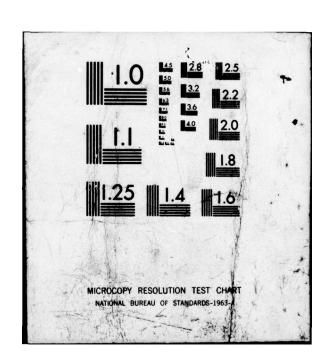






DATE FILMED

3 **-** 77



ADA 035353





UNIFIED TREATMENT OF INEQUALITIES OF THE WEIERSTRASS PRODUCT TYPE

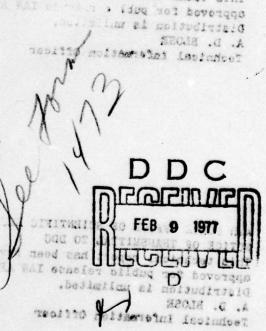
by

Emad El-Neweihi, University of Kentucky and Frank Proschan , Florida State University

> FSU Statistics Report M396 AFOSR Technical Report No. 67

Proposed in the same of the sa

December, 1976 Department of Statistics The Florida State University Tallahassee, Florida 32306

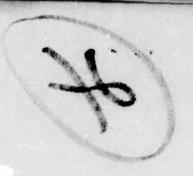


ALE PORCE CHANGE OF BULLETIE

NOTICE OF TRANSACTION TO DOC

This technical rest the Lead revoluted and to

Research sponsored by the Air Force Office of Scientific Research, AFSC, USAF, under Grant AFOSR-74-2581C. Reproduction is permitted for any purpose of the United States Government.



UNIFIED TREATMENT OF TWEOVALIFIES OF THE WEIRLSTEASS PRODUCT TYPE

Prend Ll-Neweihl, University of Kentucky and Frank Proschan, Florida State University FSU Statistics Report H396 AFOSK Technical Report No. 67

AIR FORCE OFFICE OF SCIENTIFIC NECLARCH (AFSC)
NOTICE OF TRANSMITTAL TO DDC
This technical regret has been reviewed and is
approved for public relevance IAW AFR 150-12 (7b).
Distribution is unlimited.
A. D. BLOSE
Technical Infernation Officer

erar pecember 11976
Department of Untartation
The Florida Scate University
Tallahanses, Florida 32305

AIR FURCE OFFICE OF SCIENTIFIC RELLER SOLD NOTICE OF TRANSMITTAL TO DDC
This technical report has been reviewed approved for public release IAW AFR 190-12 (12).
Distribution is unlimited.
A. D. BLOSE
Technical Information Officer

Respanch ar use.ed by the Air Force Office of Schooling Wesserch, Arequired the Air spending of the Control and Surpose, or the under Grant Arone-14-15816. Reproduction is permitted Thir any purpose, or the under Grant Green Covernment.

WW 032323

ABSTRACT

A typical inequality of the Weierstrass product type states that $\begin{array}{c} n \\ \pi \\ (1+A_1) \geq (n+1)^n \\ \pi \\ A_1, \end{array}$ where $A_1 \geq 0$, $i=1,\ldots,n$, and $\sum_{i=1}^n A_i = 1$. It is short note shows that the powerful tools of majorization and Schur functions provide a unified method for deriving a variety of Weierstrass type inequalities.

179	White Section
76	Buff Section
MAAROORCED) []
DETIFICATION	k
7	A/ATAILABILITY CODES
3 24500 3100	
	A SPECIAL



UNIFIED TREATMENT OF INEQUALITIES OF THE

(2), (3) and (4) can be obtained by a unified appreach using the powerful

Frank Proschan , Florida State University and Balling & State University

A Schur-function is a function that is monotone with respect to this

In a note by Klamkin and Newman [1], it was shown that

(4) fellow immediately by observing that certain functions are Schur-functions, thus providing a unifying and more transmarent put
$$\prod_{l=1}^{n} (l+n) \le (L^{k+1}) \pi$$
 (1) thus providing a unifying and more transmarent put $\prod_{l=1}^{n} (l+n) \le (L^{k+1}) \pi$ (1)

Section 2. We now give the standard definitions and
$$\frac{n}{r} \cdot n(1-n) \le (\frac{1}{r} - k-1) + 1 \cdot n(1-n)$$
 and Schur-functions quoded for proving inequalities (1), (2), (3) and (4).

where $A_1 \ge 0$ (i=1,2,...,n) and $\sum_{i=1}^{n} A_i = 1$. Under the same conditions it was shown by Klamkin in [2] that

(3)
$$\frac{n}{n} \xrightarrow{(1+A_1)} \frac{n}{(n+1)^n} \ge \frac{(1-A_1)}{(n-1)^n}$$

$$\frac{1}{n} \xrightarrow{(1+A_1)} \frac{1}{(n-1)^n} = \frac{1}{n} \xrightarrow{(1-A_1)} \frac{1}{(n-1)^n}$$

with equality if $A_i = \frac{1}{n}$. In [2] the author refers to a similar inequality to (2) Ky Fan [[4], p. 363] under tighter conditions on A_i but more relaxed condition on EA_i , namely,

Definition 2. A function fix + k is eatin to be a Schur-coave (Schur-coave) in
$$\frac{1}{n} = \frac{1}{n} = \frac{1}{$$

Inequalities (1), (2) and (3) are proved by the authors as extensions of the Weierstrass product inequalities [see [1]].

Research sponsored by Air Force Office of Scientific Research, USAF, AFSC, under AFOSR Grant 74-2581C. Reproduction is permitted for any purpose of the United States Government.

UNIFIED TREATMENT OF INCOUALITIES OF THE

The main purpose of this note is to show that the inequalities (1), (2), (3) and (4) can be obtained by a uniform approach using the powerful tools of majorization and Schur-functions. Majorization (defined in Section 2) is a partial ordering in R_n, the n-dimensional Euclidean space. A Schur-function is a function that is monotone with respect to this partial ordering. In Section 2 we show that inequalities (1), (2), (3) and (4) follow immediately by observing that certain functions are Schur-functions, thus providing a unifying and more transparent proof of these inequalities.

Section 2. We now give the standard definitions and results of majorization and Schur-functions needed for proving inequalities (1), (2), (3) and (4).

Given a vector $\mathbf{x} = (\mathbf{x}_1, \dots, \mathbf{x}_n)$, let $\mathbf{x}_{[1]} \ge \mathbf{x}_{[2]} \ge \dots \ge \mathbf{x}_{[n]}$ denote a decreasing rearrangement of $\mathbf{x}_1, \dots \mathbf{x}_n$.

Definition 1. A vector # is said to majorize a vector x' if

$$\frac{j}{\sum_{i=1}^{\Sigma} x_{[i]}} \ge \frac{j}{\sum_{i=1}^{\Sigma} x_{[i]}}, \quad j=1,\dots,n-1,$$

condition on IA;, namely,

United States Covernment.

and

with equality if
$$A_1 = \frac{1}{n}$$
. In [2] the suction refere by 3s statist inequality to (2) My Fan [[4], p. 363] under tighter conditions on A_1 but more relaxed

in symbols $x \ge x'$.

Definition 2. A function $f:R_n \to R$ is said to be a Schur-convex (Schur-concave) function if $x \ge x$ implies that $f(x) \ge (\le) f(x')$. Functions which are either Schur-convex or Schur-concave are called Schur-functions.

Theorem 1. Let ϕ : $R \to R$ be a log-convex (log-concave) real-valued function.

Let g: $R_n \to R$ defined by $g(x) = \pi \phi(x_1)$, where $x = (x_1, \dots, x_n)$. Then i=1 g is Schur-convex (Schur-concave).

Theorem 1 follows immediately from a theorem by Oetrowski, (1952) [3].

We are now ready to prove inequalities (1), (2), (3) and (4). Let

 $\tilde{A} = (A_1, \dots, A_n)$, where $A_1 \ge 0$, $\sum_{i=1}^n A_i = 1$. Clearly $\tilde{A} \ge (\frac{1}{n}, \dots, \frac{1}{n})$.

Now let $\phi_1(x) = \frac{1+x}{x}$, where $x \ge 0$. Then $\phi_1(x)$ is log-convex. Therefore

by Th. 1, $\pi \phi_1(x_i)$ is Schur-convex. Hence $\pi \frac{1}{1-1} \frac{(1+A_i)}{A_i} \ge (n+1)^n$ which establishes (1).

Now let $\phi_2(x) = \frac{1+x}{1-x}$, $0 \le x < 1$. Then $\phi_2(x)$ is log-convex and so by Theorem 1, $\pi \phi_2(x_1)$ is Schur-convex. It then follows that $\pi \frac{(1+A_1)}{(1-A_1)} \ge \frac{(n+1)^n}{(n-1)^n}$,

which establishes (3). Equality holds if $A_i = \frac{1}{n}$ since $\phi_2(x)$ is strictly log convex.

Now let $\phi_3(x) = \frac{1-x}{x}$; then $\phi_3(x)$ is log-convex for $0 < x \le \frac{1}{2}$ and log-concave for $\frac{1}{2} \le x < 1$. Also let $A = (A_1, \dots, A_n)$, where $0 \le A_1 \le \frac{1}{2}$. Clearly

$$A \geq (\frac{\sum A_1}{n}, \dots, \frac{\sum A_1}{n}), \text{ so that } \frac{n}{\pi} (\frac{1-A_1}{A_1}) \geq (\frac{1-\sum A_1/n}{\sum A_1/n}) = (\frac{n-\sum A_1}{\sum A_1}), \text{ establishing (4)}.$$

Finally, let $A_1^1 = ((n-1)A_1, ..., (n-1)A_n)$ and let $A_2^2 = (1-A_1, ..., 1-A_n)$ where $A_1 \ge 0$, $\Sigma A_1 = 1$. It can be easily verified that $A_1^1 \ge A_2^2$. By Theorem 1,

 $g(x) = \pi x_1$ is a Schur-concave function. It then follows that

$$(n-1)^n \xrightarrow{n}_{i=1}^n A_i = \xrightarrow{n}_{i=1}^n (n-1)A_i \le \xrightarrow{n}_{i=1}^n (1-A_i),$$

proving inequality (2).

It is apparent that many additional inequalities of the Weierstrass product type can be formulated and proved by choosing the appropriate log-concave function, forming products to obtain a Schur-convex function, and then using Definition

2 above. (A_1, \dots, A_n) , where $A_1 \ge 0$, $A_2 = 1$, Clearly $A_1 = (A_1, \dots, A_n)$.

Now let $\phi_1(x) = \frac{1+x}{x}$, where $x \ge 0$. Then $\phi_1(x)$ is log-convex. Therefore by Th. 1, $\frac{\pi}{\pi} \phi_1(x_1)$ is Schur-convex. Hence $\frac{n}{\pi} \frac{(1+A_1)}{A_1} \ge (n+1)^n$ which cataliance (1).

- M.S. Klamkin, D. J. Newman. Extensions of the Weierstrass product inequalities.

 Math. Mag., 43 (1970), 137-140.
- M.S. Klankin. Extensions of the Weierstrass product inequalities II. Amer. Math. Monthly 82 (1975), No. 7, 741-742.
- A. Ostrowski. Sur quelques applications des functions convexes et concaves au sens de I. Schur. J. Math. Pure. Appl. 31, (1952), 253-292.
- D. S. Mitrinovic. Analytic Inequalities. Springer-Verlag,

 Heidelberg (1970). O tol xavgoo-gol al (x) analytic x = (x) and you woll woll you would be a constant.

For $\frac{1}{2} \le x \le 1$. Also let $\underline{A} = (A_1, \dots, A_n)$, where $0 \le A_1 \le \frac{1}{2}$. Clearly

 $A \geq \langle \frac{n}{n}, \dots, \frac{n}{n} \rangle, \text{ so that } \frac{n}{n} \langle \frac{A+1}{A_k} \rangle \geq \left(\frac{n - 2A_k/n}{2A_k/n} \right) = \left(\frac{1-2A_k}{2A_k} \right), \text{ establishing (4)}.$

Finally, let $A^1=((n-1)A_1,\dots,(n-1)A_n)$ and let $A^2=(1-A_1,\dots,1-A_n)$ where $A_1\ge 0$, $\Sigma A_1=1$. It can be easily verified that $A^1\ge A^2$. By Theorem 1,

 $g(\mathbf{x}) = \frac{1}{1 \times 1}$ is a Schur-concave function. It then follows that

 $(n-1)^n \xrightarrow{\pi} A_{\underline{1}} = \frac{n}{\pi} (n-1) A_{\underline{1}} \leq \frac{n}{\pi} (1-A_{\underline{1}}),$

proving inequality (2).

It is apparent that many additional inequalities of the Meisretrass product type can be formulated and proved by choosing the appropriate log-concave function.

AFOSK (19) REPORT DOCUMENTATION PAGE	READ INSTRUCTIONS BEFORE COMPLETING FORM
VAFOSR - TR - 77- 0034 TR-67	NO. 3. RECIPIENT'S CATALOG NUMBER
TITLE (and Subtitle)	5. TYPE OF REPORT & PERIOD COVERED
NIFIED TREATMENT OF INEQUALITIES OF THE WEIERSTE	
RODUCT TYPE.	rest.
	S PERFORMING ONG REPORT NUMBER
AUTHOR(A)	(14) FSU-Statistic 3
1 (15	S. CONTRACT OR GRANT NUMBER(O)
mad/El-Neweihi Frank/Proschan	4F- AFOSR 12581-74
PERFORMING ORGANIZATION NAME AND ADDRESS TOTICA State University 977	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
epartment of Statistics allahassee, Florida 32306	61102F 2364/A5)
allahassee, Florida 32306	
CONTROLLING OFFICE NAME AND ADDRESS	12. REPORT DATE
Air Force Office of Scientific Research/NM	()/) Dec 76
Bolling AFB, Washington, DC 20332	13. NUMBER OF PAGES
4. MONITORING AGENCY NAME & ADDRESS(If different from Controlling Offic	e) 15. SECURITY CLASS. (of this report)
(12) n	
17/7	UNCLASSIFIED
- /-	15a. DECLASSIFICATION/DOWNGRADING
DISTRIBUTION STATEMENT (of the obstract entered in Block 20. If different	(max Banara)
. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different	from Report)
. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different	from Report)
7. DISTRIBUTION STATEMENT (of the abetract entered in Block 20, if different	from Report)
	from Report)
	from Report)
. SUPPLEMENTARY NOTES	
SUPPLEMENTARY NOTES KEY WORDS (Continue on reverse side if necessary and identify by block numbers)	·••)
DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different by Supplementary notes KEY WORDS (Continue on reverse side if necessary and identify by block numbers are supplementary product inequalities, Schur functions	·••)
SUPPLEMENTARY NOTES KEY WORDS (Continue on reverse side if necessary and identify by block numbers)	·••)
SUPPLEMENTARY NOTES KEY WORDS (Continue on reverse side if necessary and identify by block numbers)	·••)
KEY WORDS (Continue on reverse side if necessary and identify by block numbers eigenstrass Product Inequalities, Schur functions	, Majorization
KEY WORDS (Continue on reverse side if necessary and identify by block number ierstrass Product Inequalities, Schur functions ABSTRACT (Continue on reverse side if necessary and identify by block numbers	(Majorization
SUPPLEMENTARY NOTES KEY WORDS (Continue on reverse side if necessary and identify by block number in the stress of the stress o	oduct type states that
SUPPLEMENTARY NOTES KEY WORDS (Continue on reverse side if necessary and identify by block number in the strain of the strain o	oduct type states that
EXECUTION OF THE PROPERTY AND A STRACT (Continue on reverse side if necessary and identify by block numbers of the strains of	oduct type states that
EV WORDS (Continue on reverse side if necessary and identify by block numbers are supported in the support of the Weierstrass product inequality inequality inequality of the Weierstrass product inequality inequa	oduct type states that n, n, and Σ A ₁ = 1. This 1=1
SUPPLEMENTARY NOTES KEY WORDS (Continue on reverse side II necessary and identify by block number in the stress of the stress o	oduct type states that n, n, and E A, = 1. This i=1 orization and Schur functions
SUPPLEMENTARY NOTES KEY WORDS (Continue on reverse side if necessary and identify by block number of the strass product Inequalities, Schur functions ABSTRACT (Continue on reverse side if necessary and identify by block number of the Weierstrass product inequality in the Weierstrass product inequality inequalit	oduct type states that n, n, and E A, = 1. This i=1 orization and Schur functions